Whole-class discussion in the mathematics classroom: Analyzing a multimedia case in teacher education

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Abstract. This study addresses the use of a multimedia case in teacher education by mathematics pre-service teachers. Focusing on the whole-class discussion phase of an inquiry-based lesson in 7th grade, we aim to understand how pre-service teachers analyze an episode of this phase of the lesson regarding the teacher’s actions and role in conducting the discussion. We analyze the participants’ written productions while exploring the case. Data reveal that pre-service teachers can identify several central actions of the teacher in the case, both in promoting students’ mathematical learning and in managing their interactions. Data also show some understanding of the role of teacher’s questioning in the interpretation of those actions. Results suggest some issues for future research and for the exploration of the case in contexts of teacher education and professional development.

Introduction
Research has paid increasing attention to the inquiry-based approach to teaching and learning mathematics. Despite acknowledging the importance of other phases of an inquiry-based lesson, the complex role of the teacher in conducting whole-class discussions and systematizing mathematical ideas has been highlighted (e.g., Canavarro, Oliveira, & Menezes, 2012; Cengiz, Kline, & Grant, 2011; Stein, Engle, Smith, & Hughes, 2008). This paper is based on the use of a multimedia case focused on the practice of inquiry-based teaching. Drawing on a wider research, we seek to understand how pre-service teachers (PSTs) analyze one episode of a whole-class discussion in an inquiry-based lesson, focusing on the actions of the teacher who stars in the case.

Inquiry-based teaching
The practice of inquiry-based teaching steps away from a transmission perspective of the teaching and learning process (Canavarro et al., 2012). Learning is conceived as a process that is simultaneously individual and collective “based on students’ interaction with mathematical knowledge, while engaged in mathematical activity, and also on students’ interaction with other classroom members (classmates and teacher), being involved in processes of negotiation of meaning” (Oliveira, Menezes & Canavarro, 2013, p. 31). Tasks are particularly relevant as they are springboards for students’ mathematical activity. Tasks may be of different types but they must gather certain characteristics; namely, they should: cast a challenging situation; allow students to build on their experience to solve them, and to use different strategies with distinct levels of mathematical sophistication; and foster the conceptual understanding of the mathematical notions related to the knowledge students construct in the classroom.

We consider an inquiry-based lesson to be usually organized around four phases (Oliveira et al., 2013). In the introduction to the task phase, the teacher presents the task and seeks to ensure that the students understand what to do. The teacher organizes the students’ work in pairs or small groups, provides them with the necessary materials and pushes them to engage in the task. In the realization of the task phase, the teacher monitors and supports students’ autonomous work, ensuring the maintenance of the
task’s cognitive demand. The teacher poses questions, offers hints, suggests forms of representation, and asks for clarification and justification of ideas and procedures, avoiding validating students’ answers. In this phase, two actions are crucial to the success of the following phase: selecting and sequencing the students’ solutions for whole-class discussion (Stein et al., 2008). The discussion of the task phase poses many challenges to the teacher. Despite the preparation for discussion made in the previous phase, there is always room for emergent ideas. This phase goes beyond the presentation of solutions, looking for common grounds and the construction of knowledge. Questioning, pressing for explanations and underlying rationale for strategies and reasoning, pushing for comparison among solutions and discussion of solution efficacy are crucial actions of the teacher (Canavarro et al., 2012; Cengiz et al., 2011; Stein et al., 2008). Managing students’ participation, discussing mathematical ideas, regardless of their correctness or clarity, and constructing knowledge using recognizable language to students are further challenges to the teacher. In the systematization of mathematical learning phase, the teacher helps the students to “recognize the mathematical concepts and procedures involved, establish connections with prior learning, and/or stress fundamental aspects of transversal mathematical processes such as representation, problem solving, and mathematical reasoning” (Oliveira et al., 2013, p. 34). The mathematical activity goes beyond synthesizing the ideas that emerged during the discussion, systematizing them and institutionalizing (new) knowledge (Canavarro et al., 2012). Sometimes, this last phase is simultaneous to the discussion, and there may be moments of discussion/systematization during the development of the task. Figure 1 depicts the teacher’s instructional actions and respective intentions for the discussion of the task phase (Oliveira et al., 2013). The description is based on two central aspects of the teacher’s practice: promoting students’ mathematical learning and managing classroom interactions.

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<th>DISCUSSION OF THE TASK</th>
<th>Promoting the mathematical quality of students’ presentations:</th>
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<td>- Asking for clear explanations of solutions</td>
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<td>- Encouraging questioning to clarify presented ideas or</td>
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<td>- Encouraging the analysis, confrontation, and comparison</td>
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<td>the presented solutions</td>
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<td>Regulating students’ interactions in the discussion:</td>
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<td>- Stop students’ autonomous work in solving the task</td>
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<td>- Reorganizing the seats and space for discussion</td>
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<td>- Promoting attitudes of respect and genuine interest for</td>
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<td>- Justifying why some solutions are not presented</td>
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<td>- Promoting and managing students’ participation in the</td>
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<td>- Managing students’ relationships:</td>
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<td>Creating an environment conducive to presentation and discussion:</td>
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<td>- Stopping students’ autonomous work in solving the task</td>
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<td>- Identifying and discussing mathematical errors found in</td>
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<td>the different solutions presented</td>
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<td>- Promoting and managing students’ participation in the</td>
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<td>- Managing students’ relationships:</td>
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<td>Figure 1. Teacher’s actions and intentions during the discussion of the task phase.</td>
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Multimedia cases in teacher education

The use of multimedia resources, particularly videos of classroom teaching situations, has increased in popularity in various teacher education contexts. Such resources allow to virtually access the classroom and capture a large part of its vast complexity (e.g., Alsawaie & Alghazo, 2010). Inquiry-based teaching is not a commonly found practice but it is a promising approach to developing students’ mathematical power. Videos and related components for reflection about this practice may be privileged means to direct PSTs’ attention to various aspects such as students’ mathematical thinking and the role of the teacher in conducting the classroom discourse (e.g., van Es & Sherin, 2008). Within a research project in Portugal, several multimedia cases were constructed, based on a web platform, and include lesson plans, video excerpts of lessons, students’ written productions, teachers’ interviews, and related bibliography. All cases are organized
around the four phases of an inquiry-based lesson and its analysis is guided by sequential components closely linked to those phases. The case versed in this paper is built on the discussion phase of a 7th grade lesson aiming to review and consolidate the main concepts and procedures of the teaching unit on equations (Oliveira et al., 2013). The task of the lesson (appendix 1) may be seen as a problem and may be approached in different ways, as foreseen in the teacher’s lesson plan. The diversity in the task solutions favors the discussion and the attainment of the intended learning goals. In this paper, we focus only on the PSTs’ analyses of one episode of the discussion phase, presented in a short video excerpt and respective transcription. The PSTs were challenged to structure their analyses by identifying the teacher’s actions concerning the promotion of students’ mathematical learning and the management of their interactions.

Methodology
This paper stems from an experience in the methods courses of the teacher education programs offered by two universities which lasted for four and ten sessions of 2.5 hours each, respectively, in Lisbon (UL) and Porto (UP). Working in pairs or triples around the case, 21 PSTs participated in the study. The authors played the double role of teacher educators and researchers, the first one in Porto. For this paper, the data sources are the PSTs’ written productions in answering the questions posed in the web platform regarding the selected episode. The analysis of those documents was based on Alsawaie and Alghazo’s framework (2010) (built on van Es and Sherin’s (2008) work), who consider three dimensions: (1) to identify what is important in the teaching situation; (2) to interpret the teaching situation based on the knowledge of context; and (3) to establish connections among the observed aspects and broader principles of teaching and learning. We sought to understand how these dimensions, which we related to the key elements of figure 1, are evident in the work of the PSTs.

Results
The participants were asked to analyze one episode by identifying the teacher’s actions aimed at promoting students’ learning and at managing students’ interactions and the lesson – these are the actions that we consider to be important aspects of the teaching situation (resonating with the framework of Alsawaie and Alghazo (2010) and with figure 1). The episode depicts part of the discussion when the teacher asks a group of students to share their solutions, maintaining visible on the board the solution that had just been presented by another group. The teacher’s goal is to compare and contrast both solutions and to review key concepts of the unit. Both groups chose an algebraic approach (equation) to solve the problem, though referring to different unknowns.

Important aspects of the teaching situation
The PSTs found that one of the teacher’s main actions to promote students’ learning was to keep visible, on the board, the two algebraic solutions, allowing better visualization and comparison: “[The teacher] has the students facing two solutions at the same time, comparing their similar and distinct aspects (e.g., the meaning of the unknown in each equation)” (G1, UL). Some explicitly refer the teacher’s questioning and requests for explanations, emphasizing the role of these actions in fostering students’ learning: “The teacher poses several questions so that students understand the similarities and differences between the two algebraic expressions. (…) She asks (…) about the meaning of the terms in the algebraic expressions” (G1, UP). Another action emphasized by the PSTs relates to the teacher’s efforts to have the students
understanding that the unknowns used by each of the two groups represent different objects, giving rise to two distinct equations:

The teacher (...) uses the strategy of comparing two types of solutions, of two groups (...) in which one of the groups considers the unknown x to represent Lucas and the other to represent Francisca. As such, the teacher intends the students to understand that there are different paths to reach the problem solution, i.e., the two solutions are equally valid. (G2, UP)

Though not being equivalent, the two equations lead to the (same) problem solution, as mentioned by the participants: “[The teacher] discusses with the students whether the equations are equivalent, helping them to realize that, although they have different solutions (they are not equivalent), they lead to the same problem solution…” (G2, UL).

In the episode, the students’ reactions to the two equations being compared make it clear their difficulties in understanding the notion of equivalent equations. Several PSTs mention those difficulties, associating them with the teacher’s actions:

(... The teacher repeatedly questions the students about the equivalence of the two equations. Since she does not get satisfactory answers, she chooses to change her discourse, questioning the whole class about the definition of equivalent equations. When this issue is finally clarified, the teacher insists, questioning again about the equivalence of the equations written on the board. (G3, UP)

The PSTs also highlight the teacher’s actions towards managing students’ interactions, namely promoting their participation in the collective discussion, and the teacher’s questioning is also important in this regard:

[The teacher] promotes the participation of everyone, asking a distracted student to give an answer and constantly requesting other students to state their agreement or disagreement with the given answer; (...) She manages the interventions, asking students to speak one at a time; a student offers a suggestion about a solution, saying that it is going to be different from the previous one. The teacher tries to sparkle students’ curiosity, by saying: ‘Who bets a yes and who bets a no?’; a student poses a question and the teacher redirects it to the class, asking ‘Who can answer?’. (G2, UL)

Interpretation of the teaching situation

In some written productions, there are evaluative elements regarding the teacher’s actions. The PSTs make judgments of those actions, and seek to justify their opinions:

The teacher took advantage of Margarida’s solution (...), leaving it on the right side of the board, and Mariana’s (...) was written on the left side. This was an interesting option since it allowed, via algebraic representation, to compare 2 distinct processes, which resulted in distinct solutions, but which allowed to reach the answer. (G3, UL)

The teacher’s strategy of keeping the two equations on the board, visible to all students, is seen by the PSTs as important towards several goals, namely to understand that a problem may be solved in different ways (even when one resorts to the same strategy), and to realize that the two equations are not equivalent but allow to solve the problem.

The PSTs also focus on describing the teacher’s actions while interpreting those actions, taking a critical stance and even offering an alternative to those actions, though implicit.

Realizing that students are misunderstanding the attribution of two distinct values to the same unknown (...), the teacher goes further in the discussion about the classification of equations, involving several students, so that they comprehend that each equation as only one solution. However, we do not know whether this exchange of ideas was productive to the class (...). The students’ difficulty in this case dealt much more with the interpretations of each solution and its adequacy to the problem than with questions about validity of solution sets. (G3, UP)
The PSTs interpret the teacher’s care in involving the largest possible number of students in the discussion, and the press for students’ accountability in validating the mathematical ideas that emerge in the discussion:

*The teacher* asks the students to justify everything that is being written by the student who is on the board (...) When there are students on the board, the teacher moves to the back side of the classroom, which prevents her from focusing on the students on the board, involving the whole class in what is being done. (G1, UL)

The teacher’s questioning emerges again as a main engine of her actions in managing students’ interactions. The creation of a classroom environment which invites students’ participation and values the diversity of strategies or representations – for which the teacher’s questioning is a key element – are also relevant in managing interactions:

*An interesting fact is that the teacher took the opportunity to value different strategies, i.e., different paths lead to the same result, which is something very important for students to understand so that they are not afraid of sharing their ideas, which, very often, may be different, but still may be correct.* (G4, UP)

**Relationship of the teaching situation with broader ideas**

Several PSTs relate the teacher’s questioning with theoretical frames that were addressed in their methods courses. For example, they typify the teacher’s questions using Ainley’s (1988) model, and establish connections between the teacher’s actions and questions and her learning goals for this lesson:

*She poses provoking questions to stimulate students to think about the matter. With these questions and her conducting of the discussion, the teacher is able to review the topics of the unit on equations. Thus, the students talk, review the notions of unknown, equation, equivalent equations, solution set, possible and undetermined equation (...) This review exercise is of extreme importance since the task was given at the end of the unit on equations, as it helps in reviewing and consolidating important and previously learned concepts.* (G1, UP)

The teacher’s questioning, besides promoting students’ participation in the discussion, is a means to help them engaging in mathematical activities that are central in an inquiry-based lesson: the understanding of processes and procedures used by others:

*The teacher questions the students many times about their opinion regarding an answer that was given and whether they agree or disagree with their classmates’ opinions. This strategy (...) pushes students’ interactions, making them pay attention to the discussion, realizing the steps and the strategy used by the pair that is presenting and reflecting on the arguments of the students who are participating in the discussion.* (G1, UP)

The teacher’s questioning is also highlighted as impacting students’ comprehension of the key concept of unknown. Implicitly, the PSTs make references to the lesson goals:

*The teacher, more than to manage, ends up (...) involving various students in the discussion, while avoiding that they get dispersed and deviate from the essential. The teacher poses questions related to the meaning of the differences indicated by them [the students], so that they can explore the concept of unknown...* (G3, UP)

More explicitly, the teacher’s actions are associated with the development of students’ reasoning and communication, two central issues of the program goals (ME, 2007).

*The teacher promotes students’ learning by launching questions to the class (purposefully giving a wrong answer while saying that ‘the equation is undetermined’) leading the discussion towards having the students thinking a bit more about this concept. (...) In this way, the teacher helps the students developing their mathematical reasoning and communication.* (G4, UL)
Final remarks
In general, PSTs identify aspects of the teaching situation and structure their written productions around a chronological description of events. They focus the relevant aspects of the teaching situation on the teacher’s actions, especially those aimed at promoting students’ mathematical learning, in particular fostering the understanding of different strategies, contrasting them and deciding on their degree of efficacy. They also emphasize the teacher’s questioning, especially when asking students for clarification or justification of their ideas. The PSTs highlight the promotion of participation of all students in the collective discussion and the teacher’s questioning as instrumental in stimulating students to analyze each other’s ideas, even those that are not correct. Many written productions include interpretations of the teacher’s actions based on central issues of an inquiry-based lesson (Oliveira et al., 2013). Those actions are frequently judged but the PSTs seek to justify their opinions. Several PSTs make connections to theoretical elements addressed in their methods courses. Relating the teacher’s questioning to Ainley’s (1988) model is somewhat frequent, suggesting that the PSTs understand the different purposes of the teacher’s questions. Yet, they were not requested to address theoretical elements in their analyses. This might be an aspect to improve in future uses of the multimedia case. Overall, the PSTs seem to understand that the goal of the discussion phase of an inquiry-based lesson is not to correct students’ solutions but rather to stimulate and support the confrontation of strategies, which allows them to gain or consolidate knowledge, as referred by Oliveira, Menezes and Canavarro (2013). When highlighting the teacher’s questioning, the PSTs show to understand that, within an inquiry-based approach, the teacher encourages the students to go beyond one single solution method, and to think and mathematically. These aspects are mentioned by Cengiz et al. (2011). This study is focused only on the discussion phase of a 7th grade inquiry-based lesson, in teacher education. Further research is needed to understand how (future and practicing) teachers analyze other phases of the lesson, and how the work around the multimedia cases assists them in developing a global perspective of inquiry-based teaching.

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References


**Task: Election of the Class Representative**

The teacher coordinating the election of the class representative reported that:

- All students in the class voted (30 students) and that there were no null nor blank votes
- Only three students received votes: Francisca, Lucas and Sandra
- Lucas received two votes fewer than Francisca
- Sandra received twice as many votes as Lucas.

Who won the election? With how many votes?

_Do not forget to present and explain how you found the solution._