The teaching of mathematics in undergraduate (UG) secondary initial teacher education (ITE): some students' responses to enquiry based pedagogy with transformative intentions

Hilary Povey
Mathematics Education Centre, Sheffield Hallam University
h.povey@shu.ac.uk

Abstract
This paper is concerned with the teaching of mathematics in UG secondary ITE. It is argued that this context provides an opportunity for students to experience both active, engaged, meaning-making pedagogies, refereed to here as enquiry based, and to come to understand through the medium of that experience how these might link to social justice issues. The responses of a single cohort of students to a mathematics module in which such an approach was exemplified serves as a case study.

Introduction
Researchers into the teaching of mathematics in school classrooms have shown that active, engaged, meaning-making pedagogies promote learning and encourage the development of authoritative, confident learners (Mason, Burton and Stacey, 1982; Ollerton and Watson, 2001; Alro and Skovsmose, 2002). In addition, connections have been made between such pedagogies and the possibilities for social justice in mathematics classrooms (Skovsmose, 1994; Povey and Angier 2006; Gutstein, 2006; Boaler, 2008; Staples, 2008; Gates and Noyes, 2014).

Unsurprisingly, therefore, such practices are routinely advocated by mathematics teacher educators in their work with ITE students. However, as Nolan points out, in spite of this, 'traditional textbook and teacher-directed approaches still prevail in most secondary classrooms' (2012, p201). It is suggested that one important contributor to this complex phenomenon is that pre-service and in-service teachers draw on their own experience of learning mathematics rather than on their learning of mathematics pedagogy at the university (Ball, 1988; Nolan, 2012). One approach that initial mathematics teacher educators can use to counter this failure to transfer the ideas explored in the ITE classroom to secondary mathematics classroom practice concerns mathematics subject teaching in the UG years.

The focus of this paper is the impact on ITE students of studying academic mathematics as an UG subject in its own right through the medium of an enquiry based pedagogy with transformative intentions, an area that has had comparatively little research attention so far. The purpose is to consider how the enactment of this pedagogical approach in the students' own learning of mathematics impacts on students' understanding of what it is to learn - and, perhaps, therefore, to teach - mathematics.
The common sense justification for believing that the experience of learning mathematics in this way may eventually impact on teaching is that the promotion of recommended classroom practices is much more effective when that practice is not just talked about and recommended but also exemplified by the teacher educators themselves. If we experience enquiry based pedagogic practices ourselves as learners, as it were “from the inside”, we are much more likely to understand their nature, to be convinced of their merits, to begin to identify with them and to start to acquire the skills and competences necessary to author such practices ourselves. It seems a plausible conjecture that, where opportunity permits, adopting a strategy of the students themselves experiencing learning mathematics in ways that resemble those advocated will be of benefit. This paper tentatively offers evidence in support of this conjecture in examining the views of a group of pre-service teachers during their ITE. A follow-up research project is examining what, if any, is the legacy in terms of their teaching.

The study
The study reported here is based on an UG honours level pure mathematics module taken by secondary ITE students. It focuses on one particular cohort. There were 40 students in the class. About half of them were already graduates but in disciplines other than mathematics; their academic backgrounds were varied but all of them had approximately one year’s study of higher education mathematics behind them. They were now following a one year course working at intermediate and honours level UG mathematics to supplement their existing mathematical studies prior to a further year focused on mathematics professional practice. The other half were current secondary mathematics ITE UGs who had undertaken initial and intermediate mathematics modules and were now working at honours level as part of completing their mathematics-with-education degree.

The module thus sits in two courses; both involve significant work on students' mathematics subject knowledge and, broadly, espouse a learner-centred, enquiry-based pedagogy, sometimes with transformative intentions. The modules are taught within a mathematics education centre by staff who have themselves been school and college teachers before entering the university and who place considerable emphasis on the idea of meaning-making within a learning community and of the social construction of knowledge (Povey and Angier, 2007). This stands somewhat in contrast to the pedagogy experienced by many (though not all) UG mathematicians in the United Kingdom (UK). Such students are often reported as failing or disliking the subject (Macrae et al, 2003; Mann, 2003; Solomon, Croft and Lawson, 2010). In her study of mathematics UGs at an English university, Solomon (2007) found that

… these students experience mathematics as something "done to them" rather than "done by them"; they do not share in the ownership of meaning, let alone meaning making - they are excluded from that vital aspect of participation which Wenger identifies: negotiation … the
majority of students did not perceive themselves as potential negotiators or owners of meaning. (p90)

It is more common in the UK for UG secondary ITE students to be taught mathematics alongside those following a standard mathematics degree than for them to have a bespoke course. If the argument of this paper is valid, perhaps this practice needs rethinking.

This enquiry is "researching from the inside". It is best understood as a single evaluation case study, not making any claim to representativeness but rather offering a 'serious example' (Skovsmose, 1994, p9). I use interview data from students to explore their experience of and reaction to a module that I taught. I see the interviews as cultural spaces where the students, through improvisation, come to an understanding of their experiences; and where it is possible to detect if, and, if so, to what extent and in what way, the espoused theory was visible to them. The interviews also offer evidence of the extent to which they were willing to experiment with embracing aspects of that theory in their current sense of whom they themselves are becoming.

The interview data - focused on the students' experience of the module - were gathered just before the end of a semester. The taught part of the course was already finished and the formal assessment by blind-marked examination was yet to come. The interviews, each of which lasted about an hour, were conducted by an experienced researcher from another university who was unknown to the participants. I explained the purpose of the enquiry to the students in writing, making it clear that they were under no obligation to take part and could withdraw from the study at any time. I assured the students that all data would be reported anonymously unless they specifically requested otherwise. I would not be teaching them again after this module and I guaranteed that the data would not be viewed by me until after all the students had finished engagement with the module and had been assessed. I hoped in this way to address the ethical issues involved with obtaining evaluative statements from the students involved in a course I was myself teaching.

Everyone in the class agreed to be interviewed but logistical constraints on timetabling meant that only approximately half of them, eighteen in all, participated. There were fewer males than in the overall group (22% compared with 33%) and fewer representatives of the UG route (33% compared with 53%). The clusters for interview were mixed with respect to gender and the two cohorts and were not composed of friendship subgroups from the class. It is likely that the more engaged and committed students are over-represented in the sample. Nevertheless, the data provide useful evidence of the impact that enquiry-based mathematical pedagogies can have on the thinking of secondary mathematics ITE students.

We had agreed a very open semi-structured protocol for the interviews and briefed the students accordingly. The interviews were all transcribed by an independent transcriber (with occasional editing for clarity). The initial analysis, despite being careful and systematic was nevertheless iterative and therefore somewhat "messy". I worked with the transcription texts, reading
and re-reading the transcripts many times, tentatively extracting what seemed to me to be interesting and coherent passages of dialogue. I printed each such passage onto a separate sheet of paper and physically grouped, de-grouped and re-grouped the sheets, searching them for, and annotating, emergent themes. I did not identify the individual participants in the transcripts at this stage in order to gain a little distance from the texts. I then listened to all the tape recordings more than twice, attempting 'radical listening' (Clough and Nutbrown, 2012, p99), immersing myself in the full data set to gain a sense of how the emerging themes were, or were not, grounded in the data. I then repeated the initial process of working with the transcripts, checking out and supplementing the existing themes that had emerged and also looking for new ones. No new themes emerged at this stage but some were modified and / or enriched. Finally, I referred to my record of which students had participated in which group interview and listened carefully to the tapes for a final time in order to identify the participants in the extracted passages.

In presenting the themes in the writing of the paper, I mostly have opted to present sections of dialogue rather than individual comments to give some insight into the relational interconnectedness of the interviewees, in itself understood as an outcome of and evidencing engagement in the learning community.

**The module**

The module can be thought of as a specific and particular cultural world with its own 'socially produced, culturally constructed activities' (Holland et al, 1998, p41), a world of inquiry-based, collaborative, authored mathematics learning. The cultural world is not itself physical - rather it is imagined - but it 'move[s] through us as spoken discourse and embodied practice' (Holland et al, 1998, p251) and is opened by and entered through **artefacts**. These are the means by which the cultural world is evoked, collectively developed, individually learned, and made socially and personally powerful … artefacts assume both an obvious and necessary material aspect and an ideal or conceptual aspect, an intentionality, whose substance is embedded in the figured world of their use. (Holland *et al*, 1998, p61)

Thus, artefacts assume and express intentionality. Examples of the artefacts, including discursive practices, mediating this cultural world are:

- my language and the implicit and explicit rules I set up for classroom talk;
- the attractive and mathematical nature of the physical space for studying;
- the organisation of that space with seven sets of tables arranged to accommodate groups of about six;
- the requirement to work in mixed, tutor-designated groups throughout the taught section of the module;
- the tasks set;
- the pattern of attendance with fewer, longer sessions to allow for extended work on particular tasks;
- the physical objects used - large, colourful physical models, shared task recording sheets, cards to be flexibly sorted and organised rather than static texts, erasable short-term responses to group questions;
- the open book examination where students are allowed to bring in, for example, mathematical models they have made and where there are more than 100 marks with all work undertaken credited.

The emphasis throughout is on the nature of mathematical systems and structures with the intention that the varied content allows the development of such meta-awareness and understanding. The module is equivalent to one quarter of a year's study at Honours level. For almost all of the students, the teaching of the module represents 'a totally new experience' (interview data).

**The students' responses**

I now present the themes which emerged through the analysis of the interview data, clustering them as:

- achievement by and through all
- the affordance of pleasure
- a community of enquirers and problem-solvers.

These emerged clearly from all three group interviews and with a significant level of commonality in what was said. There was only one dissenting voice, Lucy, who was “not a massive fan” of the sort of practices encountered in the module and contrasted it unfavourably with her previous experience at university.

Lucy: [My previous experience is of a] massive lecture hall, 300 people. You sit in the lecture, take the notes, go to the tutorial if you want or not, but I never really had to because you just take the notes home and you’ve got it. You sit an exam that’s based on your lecture notes, bang it out and you’re done…

This is an important warning voice and, with a colleague, I have written elsewhere about such voices and what they might mean (Jackson and Povey, in preparation). Here, however, I report on the views of the rest of the students.

I consider each of the themes in turn though the reader will perceive overlaps and continuities between them.

**Achievement by and through all**

In my practice, I emphasise throughout that everyone can achieve and that the achievement of one supports the achievement of all. This is a key figuring of the cultural world which is initially performed by me through explicit ‘censuring and "extinction" of behaviours' (Holland *et al*, 1998, p173) that do not support this discourse - for example, disallowing talk about people as "clever" or "able"
or about oneself as "useless", "thick" or "stupid" - and through the use of practices drawn from complex instruction in group-work (Boaler, 2008). I frequently require that everybody in a table group must understand before the group moves on (Staples, 2008). In this, I offer a challenge to the 'almost universal fixed ability beliefs which are perpetuated by the pedagogic practices that surround [UG mathematics] and permeate the UG community of practice' (Solomon, 2007, p88). This potentially transformative practice is re-performed by the students in many instances during the interviews. For example:

Anna: I think we’re all on kind of a level as well. Like we all have obviously our strengths and weaknesses in certain areas, but they’re all different so we can actually get that collaborative kind of thing that everyone’s talking about.

Sally: … there’s always somebody in the group, whether it’s one person or a couple of people, that where they seem to be getting that bit of what we’re doing, you know, they’ll go through it again and again if need be and I’ve been in the situation where I’ve understood something and somebody else hasn’t and they’ve been getting a bit stressed out with it and I’ve been able to explain to them, you know, like taking it back a step and going through things

Ruth: Yeah, people are very good at sort of, you know, giving you the time if they can see something’s bothering you or, you know, something’s really annoying you that you can’t get. People are very good at sort of, you know, helping you through and getting everybody through at the same… not necessarily the same pace, but getting everybody to the same level before everyone moves on.

On one occasion, small group discussion of a problem in graph theory was followed by individual votes on what was the correct solution. I adopt this practice because I know that this particular problem almost always leads to an initial set of incorrect answers, thus provoking an opportunity to destabilise notions of hierarchies of “ability”. In such a context, despite initial failure, they are able to position themselves as capable and to use such an experience, which in other worlds would have the opposite effect, to re-assert this.

Matt: Through these modules [we definitely think differently about mathematics] yeah. I mean there’s one example where the vast majority of all the groups thought one particular answer was correct and then one person disagreed and that person was, you know, the one that got it right and that was a bit of a … But, you know, that’s kind of a bit of an indicator and it helps everybody else, things like that happening … Does anyone know what I’m trying to say there? … The fact that, you know, someone got it right and it’s possible to see it…

Ruth: Because you’ve got a group mentality you sort of think “One of us has got it so, you know, between us we’ve managed it and with a little bit of work it’s not out of reach for us all to,” sort of thing.
The notion of fixed ability and of limits to achievement is "common-sense" amongst mathematics teachers in England despite the fact that it is well-established that such ideas are profoundly unhelpful in supporting learners (Dweck, 2006) and operate against the interests of disadvantaged groups (Hart et al, 2004; Boylan and Povey, 2014). Experiencing a different practice allows for this taken-for-granted assumption to be unsettled in these students, themselves preparing to be teachers, and for the students for a time to see the world differently with respect to ability thinking.

**The affordance of pleasure**

A recurrent theme emerging from the interviews was the positive affective response to the experience offered in the module. This pleasure relates in part to how the mathematics itself is configured:

Mary: [The mathematics] is more interesting.

Rona: ... It feels a bit more fun. I’m not saying it’s really fun, but... [laughter]... Do you know what I mean? It just feels a bit more fun compared to doing it at school... I suppose it’s the group-work that makes it different more than anything.

Ruth: It’s been really, really good fun and I think a lot of us have had a lot more fun with maths than we’ve ever had before.

Beth: Yeah.

Ruth: ... And it’s really changed everyone’s opinion on how much fun maths can be compared to the experiences they’ve had before.

I conjecture that this owes as much to the absence of the characteristics of the conventional UG mathematics classroom - large depersonalised lecture theatres (Solomon, 2007), no fuzziness, no experimentation, no room for a personal point of view (Rodd, 2002), rather, a competition that you train for (Mann, 2003) - as it owes to the presence of the artefacts of the mathematical world they are being invited to enter - posters, colourful models, the discourse of collaboration, the physical arrangement of the classroom and the requirement to work in designated groups. These also impact on the social interrelationships that the figured world affords, creating pleasure there too.

Anna: It encourages a different way of working I think and, you know, you don’t get the chance to work in a group very often normally for maths. It’s seen as a very solitary subject, but I think it makes it a lot more interesting and a lot more enjoyable when you can sit and work with people on a subject and on a problem particularly.

Will: It’s really nice actually like being able to get to know everyone. Yeah, I do like it a lot better.

Lucy: Yeah, it’s a lot different way of working.

Will: It’s more like a little family, ain’t it?
Dora: Yeah.

A further aspect of this world, that finding things hard is to be welcomed rather than avoided, also affords a positive affected response. From the beginning, it is emphasised that the mathematics contained in the module is likely to be challenging and that this is appropriate and not to be avoided, that grappling together with things lying at the limit of our capabilities and just beyond can be productive for learning.

Matt: I think they set the stall out really well from the start ... saying “If you're finding it hard it's because it is hard, so don’t worry about it,” kind of thing. I think that’s helped a heck of a lot ... It sounds obvious, but “If you think it’s going to be hard that's because it’s hard.” “Right, well that’s good then. Fine.” So it helped me...

Beth: I think you get more of a sense of achievement at the end of it as well because if you’re told “Yeah, this is a really hard problem. It’s going to take you some time to do it. You’re going to need to work as a group, you know, between you to get it done,” and when you do get it you think “Wow, look at that! I’ve achieved that,” you know. I think it’s a really nice boost and you feel like you’ve achieved something together ...

It is not uncommon for those entering secondary mathematics teaching to reject explicitly the idea that mathematics and mathematics classrooms can give pleasure to learners (Jackson, 2011). Whilst experiencing mathematics as enjoyable is not enough to convince pre-service teachers that school mathematics can be enjoyable, it does at least provide an opportunity for reflection and questioning of school mathematics as inherently tedious and off-putting.

A community of enquirers and problem-solvers

Central to the practices of the module are the more open, group worthy tasks which position the students as a community of enquirers and problem solvers; and they come to perform the skills and competences which that positioning demands.

Dora: I think a lot of the tasks though because they’re quite open ended, it’s not stuff you can do individually. You have to do it with other people’s inputs, so you’re using other people’s ability. So the task is quite broad at first and then we kind of try to narrow things down. So it’s not one where if you wanted to you could just sit there and just work through questions on your own. It’s quite open ended, so at first it got us kind of gelling together and then we just naturally did it I think afterwards ...I think you’d have more opportunity to do it on your own [if it wasn’t open questions].

Sara: Yeah because you constantly have to ask other people and check that what you’re doing’s right and sort of agree.

These group-worthy tasks and the discourse of collaboration are used by the students to position themselves as agentic in the face of doubt or difficulty.

Sally: I think it’s a lot more open to ask questions.
Tess: Yeah.

Beth: I think if you’re sat by yourself doing it you kind of feel a bit more awkward about coming away and having to disturb someone else and ask them “I’m a bit stuck with this. Can you give me a hand?” whereas if you’re in a group if you discuss it together, not only do you get loads of people’s different opinions and views on how to do it differently, but it’s a lot easier to ask someone if you’re a bit puzzled by something “Can you explain it to me?”

Olga: And you always have that extra help from your peers. You’re not stuck and stay there being stuck.

Beth: ... Yeah, I think it definitely encouraged me to keep going with problems when I’ve been stuck and if I was working on my own I probably would have given up and moved on to something else, but when you’re in a group and you see that other people have understood it, you know that it is possible and it just pushes you to keep going that bit further.

Such a sense of personal agency and authority was found to be a central characteristic of those mathematics teachers who were able to interrogate taken-for-granted practices in schooling and work for progressive change (Povey, 1997).

Equally important in establishing a sense of authority is another aspect of the module practice: the role that answers play. This is culturally constructed in opposition to the role they usually occupy in teaching and learning mathematics where the subject is presented as a finished body of knowledge and mathematicians as never stumbling down blind alleys (Solomon, 2007, p91).

Julia: … she asks us to convince each other what the right answer is.

Tom: She doesn’t tell us straightaway, does she?

Julia: Yeah, she rarely actually gives us answers. You know, we have to work to the answers between us until we’re all happy that we have got the answer.

Int: And does it ever work that, you know, like four of you think you’ve got it and then one of you turns it round and…?

Julia: Yeah that can happen, but then you just have to go back to the beginning and really work through it.

Will: That’s happened a lot.

Sara: Yeah and someone’ll ask a question and it throws everything, but that’s quite good because then you look at it from different… you have to go through it again and really understand it.

Julia: ... Yeah, or sometimes you think you’ve understood something and then somebody can put it a different way and you’re like “Oh, maybe I didn’t understand that.”
Dora: ... I feel it almost takes away that kind of elitist side of maths where somebody will be like “I have the answer. You don’t have the answer,” because it doesn’t particularly work like that because we’ve all been discussing. Nobody really knows who’s got what, but we’ve all gained in general.

This offers a significantly different construction of what is involved in learning mathematics, emphasising that the conviction and authority of the learner, developed through interaction with others, is central to the process of coming to know. Although they sometimes find the practice frustrating, it positions the learners as capable and authoritative.

Almost all the students interviewed grew their identification with the cultural world offered by the practices through continued participation in the shared organisation of that world’s activities (Holland et al, 1998, p41). In a pattern reminiscent of the students Solomon researched (2007, p430), these students sought out continuing opportunities to work together outside the specified context of the module.

Ruth: There’s a lot of us who will come into uni when we haven’t got set lectures or come in early when we’ve got them in the afternoon because, you know, we want to sit and work as a group of us rather than sitting at home doing the work, you know, because actually we find it a lot more beneficial to sit and work as a group instead.

Int: Would you have done that before?
Rs: No. [several say no].
Olga: No, never.

The learning community had become theirs.

**Conclusion**

In this paper I have offered an account of the responses of a single cohort of students to an UG mathematics classroom in which I attempted to use an enquiry based pedagogy and where my intentions were informed by social justice issues. I have argued that the practices employed made it possible for them to challenge conventional understandings of “ability” hierarchies; to find pleasure in mathematical engagement; and to experience themselves as enquirers and problem-solvers within a learning community. This led them to find spaces and places of their own to continue to experience a group-focused approach to learning mathematics. They have had the experience of enacting engaged and participatory mathematical selves, offering the potential to undermine taken-for-granted aspects of “usual” school (and UG) mathematics such as there being an intellectual hierarchy amongst learners, mathematics being hard and cold and the learning of mathematics being an isolated and isolating experience.

My intention is that this paper should contribute to an understanding of how things might be different, to be persuasive, to be purposive, to articulate my position in the field (Clough and Nutbrown, 2012, p198). The study is an interested one, motivated by a belief that new (and more just) cultural worlds
can be created by people working together. It presents my living theory and its validity is therefore ‘tested against other people’s experience’ (McNiff and Whitehead, 2010, p253): it will only be authenticated as valid if it influences the practices of others (Ellis, Adams and Bochner, 2011).

References


