Mathematics, the first step in the evolution of understanding Physics: A preliminary investigation.

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Abstract
Arnold Sommerfeld, a German theoretical physicist, stated ‘If you want to be a physicist, you must do three things—first, study mathematics, second, study more mathematics, and third, do the same’ (as cited in Kevles, 1995, p. 200). The importance of mathematics appears to be lost in a modern world of open universities with an emphasis on social inclusion. As governments encourage universities to increase their enrolments and admit greater numbers of non-traditional students, enabling (bridging) programmes are assisting students to gain the prerequisite knowledge required for successful undergraduate studies.

This paper will present the preliminary results of a study examining students enrolled in an introductory physics course as part of an enabling programme in regional Australia. In particular the focus of the study is to determine the effect prior mathematics courses have on learning outcomes for these students. Students enrolled in the Introductory Physics course are required to have a very basic level of mathematics but mathematics and physics staff believe the level should be higher. Unfortunately, decisions are made at a higher level; as with many policy decisions made, they are made without discipline knowledge or input.

Introduction
Students habitually have a disconnect in their thinking about mathematics and physics. They often don’t relate the two; believing you have to be ‘Albert Einstein’ to be able to do physics. Arnold Sommerfeld, a German theoretical physicist, stated ‘If you want to be a physicist, you must do three things - first, study mathematics, second, study more mathematics, and third, do the same’ (as cite in Kevles, 1995, p. 200). The importance of mathematics appears to be lost in a modern world of open universities with an emphasis on social inclusion. Though studied as a language of beauty, structure and limitless imagination, mathematics in its basic form is a tool. The tools provided through mathematics are applicable to all aspects of life and disciplines of study.

The Australian government is encouraging universities to increase their enrolments and admit greater numbers of non-traditional students. As a result Australian universities have relaxed their entry requirements; reducing the necessity for students to complete traditional science subjects (Lyons & Quinn, 2010). The Australian Academy of Science identifies these changes as ‘one of the key contributors to declines in mathematics enrolments at the senior high school level’ (Lyons & Quinn, 2010, p. 109). Students are completing high school unprepared for their tertiary studies, increasing the burden on universities to provide bridging and remedial courses (Falkiner, 2012) or provide extra academic assistance.

This study examines students enrolled in Introductory Physics as part of an enabling (bridging) programme in regional Australia. In particular the focus of the study was to determine the effect prior mathematics courses had on learning outcomes for these students. Students enrolled in the Introductory Physics course are required to have a very basic level of mathematics but mathematics and physics staff believe the level should be higher.
Unfortunately, decisions are made at a higher level; as with many policy decisions made, they are made without discipline knowledge or input.

Mathematics and science education in Australia

Mathematics and science education in Australia is a bleak picture of decreasing school and university enrolments, poor supply of qualified school teachers and declining attainment relative to other countries (Dow & Harrington, 2013). Dow (as cited in De Laeter, 1989) pointed out that the number of Australian students studying physics and chemistry as a proportion of the secondary school cohort was steadily diminishing. Dekkers and DeLaeter (as cited in Hackling, Goodrum, & Rennie, 2001, p. 10) indicate that between 1980 and 1998 the year 12 cohort had increased in size by 99%; however, science subject enrolments have only increased by 31%.

Falkiner (2012) found recent enrolment records for Australian schools were not reliable and has therefore used completion records to examine the health of mathematics and science education in Australia. In 2005 students completing Year 12 mathematics accounted for 20.8% of the total number of senior completions (n = 954 937) (p. 8). By 2010 this had risen to 21.4%, though more students were opting to study the lower levels of mathematics (p. 8). Completion data for science portrayed consistent rates (14.5% to 15%) between 2005 and 2010 (p. 9). Of the students completing science, approximately 20% study physics (p. 9). This is approximately 2.9% of the competing students in 2005 to 3% in 2010.

Hackling et al. (2001) found there was a move within the national high school curriculum to focus on outcomes rather than the content which should be covered which is not preparing students for the future. They further highlight the connection between the quality of teachers and the quality of learning outcomes. Disenchantment with science has resulted in decreasing numbers of students studying science and therefore less new science teachers entering the profession (Hackling et al., 2001).

Out-of-field teaching

Further impacting mathematics and science education is the number of teachers teaching into a field that they do not have the qualifications to teach (out-of-field teaching). Data on teacher supply and demand indicates out-of-field teaching will continue to increase due to poor attraction and retention of teachers (Hobbs, 2012). On average, 16% of science teachers and 24% of mathematics teachers, nationally, are teaching out-of-field with these percentage increasing in rural and regional Australia (Hobbs, 2012, p. 21). Panizzon, Westwell, and Elliott (2010, p. 23) found, in South Australia, 42% of senior physics teachers are unqualified to teach the subject. These results are consistent with the national result of 43% in 2005 as reported by Harris, Jensz and Baldwin (as cited in Panizzon et al., 2010, p. 23). The South Australian data also revealed a higher percentage (63%) of physics teachers less than 40 years-of-age were out-of-field teaching (Panizzon et al., 2010, p. 23). This sets the stage for crisis as the over 40 year-olds reach retirement.

Within Australia obtaining a teaching degree and teacher registration enables you to teach regardless of subject qualification. We question the wisdom in this policy and believe that it is the subject knowledge that is of greater importance. Professional development for out-of-field teachers will assist in bridging subject knowledge but allowing mathematicians and scientists to teach without teaching degrees may be a more sensible solution to the problem. It is after all the in-depth knowledge of mathematics and physics and their interrelationship that enables correct succinct explanation of their concepts at all levels.
Relationship between mathematics and physics

Physics is the study of natural phenomena, requiring two methods of progression: ‘(1) the method of experiment and observation, and (2) the method of mathematical reasoning’ (Dirac, 1939, p. 122). The interaction between mathematics and its description of the universe run so deep that it can only be appreciated through thorough examination of the various facets (Dirac, 1939). If the relationship between mathematics and physics is unclear for teachers, it is probable that students will also fail to comprehend their true nature (Pereira de Ataide & Greca, 2013). Physics and mathematics may be employed in a more meaningful way if there is an appropriate appreciation of the relationship between physics and mathematics (Po-Hung & Shiang-Yao, 2011). A study conducted by Pereira de Ataide and Greca (2013, p. 1415) revealed ‘a close relationship appears to exist between the way students solve the problems and the epistemic view that students hold of the role played by mathematics in physics’. Therefore it is important to develop students' abilities to recognise and apply mathematics in a context outside of mathematics (Po-Hung & Shiang-Yao, 2011) as well as how to read physics equations so that they are not viewed as a collection of mathematical formulae that makes little or no sense (Pereira de Ataide & Greca, 2013).

Introductory Physics

The Introductory Physics course is offered as part of the Skills for Tertiary Education Preparatory Studies (STEPS) programme. This programme is available to Australians over the age of seventeen who have not met their educational goals through traditional means and are seeking an alternative entry into university. Courses/subjects within the STEPS programme are designed to provide students with the required prerequisite knowledge for undergraduate study, covering the areas of academic communication, mathematics, sciences, computing and study skills. There are three levels of mathematics: Fundamental Mathematics for University, a course in elementary mathematics, including basic algebra and linear equations; Intermediate Mathematics for University, a pre-calculus course; and Technical Mathematics for University, a course with calculus designed to meet the Engineering prerequisites.

The Introductory Physics course was initially a cut down version of the first year undergraduate physics course and taught out of the School of Engineering and Technology. It contained large amounts of reading, with few worked examples and limited exercises to practise. Students struggled to understand the content of the physics or the mathematical application. Comments from students and course evaluations highlighted the difficulties students were having and the requirement to rewrite the course.

Once the course was taken over by STEPS, it was rewritten to follow a low cognitive load format. For each concept there is a concise explanation followed by an example and real world application and then, in-line with cognitive load theory (Van Merrienboer & Sweller, 2005), the student is required to complete exercises enabling knowledge to be committed to long term memory. All exercises have fully worked solutions, enabling students to identify errors and correct their mistakes. This encourages self-directed learning. Each concept is complimented by an instruction video, facilitating a multisensory approach by enabling students to see, hear and do. All videos contain exercises for the student to complete and can be stopped and rewound or replayed at any point, a feature that students find extremely beneficial for their learning.

This course was introduced in term 2, 2013. Topics covered in Introductory Physics include measurement, kinematics, forces, work and energy, properties of matter, heat, electricity,
magnetism and electromagnetism, optics, and atomic physics. Even though Introductory Physics has been designed to have limited effect on cognitive load, teaching staff recommend that students complete Fundamental Mathematics for University and Intermediate Mathematics for University first. It is believed that student satisfaction is increased and frustration is reduced by successfully completing Intermediate Mathematics for University prior to Introductory Physics.

**The study: preliminary results**

A study is currently being conducted to survey students who have completed, are completing or will complete Introductory Physics from term 2, 2013 until term 1, 2015. Preliminary results from the study include 18 students from terms 2 and 3 (3 and 15 respectively) of 2013. The survey included a variety of question types including 5 point likert scales and open ended questions. Student enrolments and grades from these terms have been examined for correlation between subjects studied and success in physics.

In term 2, 2013, the first term of the new course, 72 students were initially enrolled in the Introductory Physics course. Of these students 33 (45.83%) dropped the course before census and 6 (8.33%) students withdrew later. In term 3 2013, 195 students were initially enrolled in the course. Of these students 119 (61.03%) dropped the course before census and 9 (4.62%) students withdrew later. The large number of students withdrawing from the course pre-census, many withdrawing before day one, is principally due the STEPS enrolment process and not the course itself. Due to the nature of the process, which involves manually enrolling students into all courses for the programme at the commencement of study, it was hard to obtain any statistically significant data from the pre-census withdrawals. A manual count of withdrawals between day one and census date was used to examine students with co-enrolments in Introductory Physics and Intermediate Mathematics for University. Absent fails (assessment not completed) and withdrawn fail were used to provide an indicator of student satisfaction and ability.

**Discussion**

The percentage of students surveyed completing Intermediate Mathematics for University prior to Introductory Physics was 72.22% (n = 13). Actual student data reveals only 51.33% (n = 58) of students passed Intermediate Mathematics for University prior to attempting Introductory Physics. Even though the structure of the course lends itself to ease of understanding and subsequent passing, the absence of prerequisite mathematics increases the cognitive load and increases the amount of time generally required to understand the concepts and mathematical applications. Regrettably the advice of mathematics and physics staff has been ignored and students are enrolled in Introductory Physics without having previously studied mathematics. All surveyed students having completing Intermediate Mathematics for University prior to Introductory Physics indicated completing the mathematics subject first benefitted their understanding of physics. Passing Intermediate Mathematics for University did not guarantee a pass in Introductory Physics; 81.03% (n = 47) of students that had passed Intermediate Mathematics for University obtained passes in Introductory Physics. Of those 18.97% (n = 11) who failed Introductory Physics but passed Intermediate Mathematics for University, all were either absent fails, withdrawn fails, and/or also failed other courses that term, indicating the reason for failing was most likely due to motivation and commitment, not the understanding of the concepts. Only one (0.88%) student attempting Introductory Physics after obtaining a fail in Intermediate Mathematics for University, achieved a pass in
Introductory Physics. This is possibly due to an increase in student motivation after having a previous fail.

Over the two term period, 31.86% (n = 36) of students studied both subjects in the same term. Any of these students receiving an absent fail, a withdrawn fail or a fail for Intermediate Mathematics for University also received the same grade for Introductory Physics. These accounted for 57.14% (n = 8), 26.67% (n = 4) and 50% (n = 1) of the total (see Table 1) absent fails, withdrawn fails and fails respectively. Given that there is a one-to-one correlation between failing Intermediate Mathematics for University and failing Introductory Physics when studying in the same term, it is strongly recommended that a pre-calculus course be completed prior to attempting to study physics. Of the students responding to the survey, 22.22% (n = 4) had studied both Intermediate Mathematics for University and Introductory Physics in the same term. One student felt they would have benefited from completing Intermediate Mathematics for University before Introductory Physics, while of the 3 responding negatively, one received an absent fail for both courses. To account for students studying the Introductory Physics and the Intermediate Mathematics courses simultaneously, Introductory Physics was scheduled/designed so that any mathematics required (eg simultaneous equations, geometry, trig, logs etc) was studied in Intermediate Mathematics for University at least one week prior to requiring it in Introductory Physics.

Despite passing the Introductory Physics course, the only student to complete the survey that had not studied Intermediate Mathematics for University felt they would have benefited from completing Intermediate Mathematics for University prior to Introductory Physics. In the two terms examined, 10 (8.85%) students had only completed Fundamental Mathematics for University; only half (n = 5) of these students passed Introductory Physics. Interestingly, only one of the 6 (5.31%) students not having studied any enabling mathematics failed Introductory Physics. Three of the students not having studied any enabling mathematics and passing Introductory Physics had studied undergraduate mathematics courses and were studying Introductory Physics in order to change career paths. The other two had recently completed high school calculus equivalent courses. It is believed that the prior mathematical content knowledge of these five was invaluable in them passing the course.

Table 1: Pass rates

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<thead>
<tr>
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<th>Term 2 2013</th>
<th>Term 3 2013</th>
<th>Total</th>
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<tbody>
<tr>
<td></td>
<td>Percentage</td>
<td>Percentage</td>
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</tr>
<tr>
<td>Number</td>
<td>Number</td>
<td>Number</td>
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<tr>
<td>Pass</td>
<td>71.79%</td>
<td>72.97%</td>
<td>72.57%</td>
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<tr>
<td></td>
<td>28</td>
<td>54</td>
<td>82</td>
</tr>
<tr>
<td>Fail</td>
<td>0%</td>
<td>2.70%</td>
<td>1.77%</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Absent Fail</td>
<td>12.82%</td>
<td>12.16%</td>
<td>12.39%</td>
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<tr>
<td></td>
<td>5</td>
<td>9</td>
<td>14</td>
</tr>
<tr>
<td>Withdrawn Fail</td>
<td>15.38%</td>
<td>12.16%</td>
<td>13.27%</td>
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<tr>
<td></td>
<td>6</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>39</td>
<td>74</td>
<td>113</td>
</tr>
</tbody>
</table>

Of the 8 students achieving High Distinctions (HDs) in Introductory Physics, 5 also attained HDs in Intermediate Mathematics for University in a prior term. The remaining 3 studied Intermediate Mathematics for University in the same term receiving grades of Pass, Credit and Distinction. But all had received high grades in all other subjects they had enrolled in, indicating a high level of commitment.

There were 14 students that were enrolled in both Intermediate Mathematics for University and Introductory Physics that withdrew from Introductory Physics between day one and census date. Seven of these students also withdrew from Intermediate Mathematics for University in the same time period. Three failed Intermediate Mathematics for University and
4 passed Intermediate Mathematics for University. This would appear to indicate that for these students the cognitive load of doing both subjects in a single term is too great.

A downfall of the study is that it only surveys students that had completed Introductory Physics and did not question students about their engagement or motivation. Therefore, valuable information, regarding why students receive absent fails or withdrawn fails, is lost. It cannot be fully determined if ability or motivation is the causative factor resulting in fails. Though, why students don’t engage with assessment is a question plaguing education the world over. In view of this, changes are being made to the study to include students that have not successfully completed the course to capture information relating to absent fails and withdrawn fails. It is further envisaged that this will also capture information on student motivation.

*Future directions and strategies*

Though, it would be preferable for all students to complete and pass Intermediate Mathematics for University prior to being allowed to enrol in Introductory Physics, this decision is not within the capability of the course co-ordinators. The course developers have instead investigated the possibilities of embedding diagnostic testing and enabling mathematics into the Introductory Physics course.

The embedding of enabling mathematics into courses can assist students without the prerequisite mathematics to gain the required knowledge. The ability of students to bridge the knowledge gap is often dependant on the size of the gap. The use of early diagnostic testing can be used to ascertain the students initial knowledge and provide strategies to improve, if required, though there have been mixed reactions from researchers (EDWARDS, 1997; Ní fhloinn, Bhaird, & Nolan, 2014) to the use of such testing. Hayes and Adams (2012) reported on the successful embedding of preparatory mathematics and diagnostic testing into undergraduate mathematics courses. Students with very low scores from the diagnostic tests were advised to enrol in an enabling mathematics course while those with small gaps were provided with material directed at the specific unknown concept. These students were also directed to obtain assistance from the Mathematics Learning Centre.

In an attempt to improve students’ understanding of physics, and building on the idea of embedding remedial assistance whilst improving students’ self-directed learning, on-line quizzes with video support are being incorporated into the Introductory Physics course. This will enable students to self-test both their physics and the required mathematics knowledge through on-line multiple choice quizzes. The advantage of this is that students are able to obtain instant feedback. The disadvantage is that students’ are only marked on their answer and not their working, which is a vital part of learning both mathematics and physics. In an attempt to overcome this and provide useful feedback, video solutions demonstrating the correct answers will run whenever an incorrect answer is selected. These videos utilise the Tablet PC and Camtasia® to enable the student to see and hear the solution unfold (Adams, Elliott, & Dekkers, 2010). Once they have watched the video, students will be given the opportunity to complete a similar question, thus completing the feedback loop. This strategy should be fully implemented by the end-of-the-year and results of the implementation available for publication the following year.

**Conclusion**

Australian government policies have resulted in universities relaxing their entry requirements. The effects of which are students opting to study lower levels of mathematics
and less science at high school and being underprepared for undergraduate study (Falkiner, 2012). Even though enabling programmes are assisting to bridge the gap and allow students to gain the prerequisite knowledge, course advisors are often ignorant of the relationship between mathematics and physics. This may result in students being inadequately mathematically prepared to study physics.

Preliminary results from a study currently surveying Introductory Physics students indicated the importance of the relationship between mathematics and physics when studying physics. Students appeared to perform better when completing Intermediate Mathematics for University prior to Introductory Physics as opposed to simultaneously. Whilst a strong correlation exists, passing Intermediate Mathematics for University does not guarantee the student will pass Introductory Physics; however failing Intermediate Mathematics for University nearly certainly predicts the student will also fail Introductory Physics. All students achieving HDs in both courses had studied them in separate terms, indicating if the two subjects are studied in the same term, one course suffers due to the increased cognitive load.

Although, providing students with an appropriate mathematical basis prior to the study of physics can assist in student achievement, failure due to motivational and commitment issues remain a concern. In view of this the researchers are amending the study in an attempt to capture this information.

References


