EFFECTS OF MATHEMATICAL MODELING INTERVENTION PROGRAM ON CREATIVE THINKING ABILITIES

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ABSTRACT
Current paper explores the effects of the model-eliciting activities (MEAs) intervention program on students’ Creative-Thinking abilities using mixed-methods realistic approach. TTCT pre- and post-tests were administered to investigate the differences between treatment and control group. Qualitative procedures were used to analyse videotapes, classroom observation and modeling products, obtained from students participating in the program. The findings indicate that the intervention program significantly improved treatment group Creative-Thinking abilities. Qualitative analyses generated 3 core categories—appropriateness, ‘mathematical resourcefulness’ and inventiveness—that help us not just to identify the Creative-Thinking abilities but also, allow us to understand how these abilities manifested themselves at the mathematical modeling creative process that had been encouraged through MEAs program.

INTRODUCTION
The knowledge revolution and the impressive technological innovations that characterize today's world require the facilitation and development of “the innovators of tomorrow who can lead the way forward” (National Science Board, 2010, p.7). In line with this, educators and researchers are still investigating how the educational system can identify, promote and develop students’ innovative and creative potential (Sriraman, 2009; National Science Board, 2010; OECD, 2013).

Model eliciting activities (MEAs) give the student opportunities to deal with non-routine, open-ended “real-life” challenges. These authentic problems encourage the student to ask questions and be sensitive to the complexity of structured situations, as part of developing, creating and inventing significant mathematical ideas (Lesh & Doerr, 2003; Amit & Gilat, 2012; Gilat & Amit, 2013). The assumption that development of creativity goes “hand-in-hand” (National Science Board, 2010, p.20) with its identification served as the focal points for our research questions:

(1) To what extent, if at all experience in eliciting mathematical models for "real-life" situations develops and improves students Creative-Thinking abilities in talented and gifted students?
(2) What cognitive abilities applied and activated by students during creative modeling processes for “real-life” situations?

CREATIVITY AND MATHEMATICAL MODELING
The following review is organized around the creative process, abilities and product (Guilford, 1950, 1967; Sternberg & Lubart, 1999; Sriraman, 2009). Guilford (1967) described the creative process as a sequence of thoughts and actions resulting in a novel production, and defined creativity as divergent thinking with its four mental abilities: fluency, flexibility, originality, and elaboration. This definition, and the scoring of these four components, has served as the basis for many creativity tests (Guilford, 1967; Torrance, 1974; Sternberg and Lubart, 1999). Torrance (1974) developed the Torrance Test of Creative Thinking (TTCT) based on Guilford’s definition of divergent thinking. In the present study, we used the TTCT-Figural to measure participants’ creativity. According to Kruteskii (1976), mathematical creativity appears as flexible mathematical thinking which involves “switching from one mental operation to another.
qualitatively different one” (p. 282), and depends on openness to free thinking and exploration of diverse approaches to a problem. Sriraman (2009) revealed the common characteristics of mathematical creativity through the Gestalt model of the creative process, defining mathematical creativity as the ability to produce a novel or original solution to a non-routine problem. Sternberg and Lubart's (1999) widely accepted definition asserts that creativity is "the ability to produce work that is both novel and appropriate" (p. 3). Mathematical MEAs provide the student with opportunities to deal with non-routine "real-life" challenges. These activities are designed according to six principles: reality, model construction, self-evaluation, documentation, sharability and reusability, and an effective prototype (Lesh & Caylor, 2007). This thoughtful design not only engages students in multiple cycles of modeling development in which they are given the opportunity to construct powerful and creative mathematical ideas relating to complex and structured data (Gilat & Amit, 2012). It also allows following students’ thinking and pattern of reasoning and requires students to represent a general way of thinking instead of a specific solution for a specific context.

**METHODOLOGY**

This study made use of mixed-method realistic approach (Creswell & Plano Clark, 2011) involving both quantitative and qualitative analyses to answer the above-defined questions. The research was conducted with 71 "high-ability" and mathematically gifted students in 5th to 7th grades who are members of the "Kidumatica" math club (Amit, 2012) and included two groups control and experimental. The experimental group participated in the intervention program, while the control group did not. The program lasted one academic year and applied in weekly 75-minute meetings. Before and after the intervention program, a TTCT pre-test was administered to students in both groups to assess their level of Creative-Thinking abilities. The program included four workshops based on different MEAs reflecting “real-life” situations, which were worked on by small groups of 3–4 students. Each MEA workshop had three parts: a warm-up activity, a modeling activity and a poster-presentation session. Upon completion of the program, both groups were evaluated with a TTCTpost-test.

**Instrument**

This study utilized the standardized TTCT-Figural (Torrance, 1974), which displays adequate reliability and validity (Kim, 2011) as a measure of creativity. The TTCT is one of the most commonly used measures of creativity in education and educational research (Kim, 2011). The test has two forms: A and B. The two forms of the figural test were used as pre and post-tests, respectively, and were scored according to the Streamline Scoring Procedure (Torrance, 2008).

**Data Sources and analysis**

To answer the first question we used the Statistical Package for Social Sciences (SPSS; 1998) to analysed the results of the TTCT-Figural pre- and post-tests from the control and experimental groups by performing repeated measure ANOVA with post-hoc analysis using Bonferroni correction.

To address the second research question qualitative procedures were used to analyse data obtained from the 47 treatment group’s students. The data includes: (1) the students' products, i.e. written documents such as mathematical models, poster presentations, letters to the hypothetical client and drafts, (2) video-recordings of the modeling sessions and of students' oral presentations, interviews (performed while students were working on their models in groups and during their model presentation), and (3) classroom observation. The qualitative analysis relay on principles derived from analytical induction (Patton, 2002; Sriraman, 2009; Marshall & Rossman, 2011) as well as techniques suggested by Strauss and Corbin (1998); Where “researcher is
guided by initial concepts and developing understandings that she shifts or modifies as she collects and analyzes the data” (Marshall & Rossman, 2011, p.208), until a coherent interpretation is obtained and all three categories and subcategories were generated and defined.

**Finding and Results - Quantitative analysis**

The results indicated significant differences between post- and pre-tests—F (1, 69) = 30.84, p < 0.000, η² = 0.31—indicating that both the control and experimental groups had improved over the course of the experiment. Moreover, results indicated a significant interaction between the groups and the time (pre-test to post-test)—F (1, 69) = 9.15, p < 0.003, η²= 0.12—indicating a significant difference between the experimental group's improvement and that of the control group. Post-hoc tests were conducted to evaluate pairwise differences among the means. In the pre-test, the mean score of the experimental group was 41.28 (SD = 22.9) and that of the control group, 42.5 (SD=24.5); in the post-test, the respective mean scores were 67.11 (SD = 24.59) and 50.08 (SD = 28.37) (Figure 2). This indicates that although both groups started with almost the same creative potential according to the TTCT-Figural, after the MEA intervention program, the experimental group exhibited higher improvement than the control group.

**Finding and Results - Qualitative analysis**

The qualitative findings provided further insight related to the mathematical creative-thinking abilities that contributed to, and constituted the creative modeling process and its significant outcomes. Three categories and subcategories were formulated with respect to theoretical framework and empirical data: mathematical appropriateness consisting of three subcategories: knowledge, documentation and utility; mathematical resourcefulness involving fluency, flexibility and elaboration, and inventiveness or originality.

Examples illustrating the meaning of the categorization are given using research data from one group of 6th-grade students’ MEA which was considered as showing the best understanding. This MEA was based on the “Bigfoot” modeling task of a “real-life” situation (Lesh & Doerr, 2003) which required students to develop a conceptual tool that would enable estimating an individual's height. Students received a cardboard with an image of an authentic large footprint's stride (Figure 3) and a measuring tape.
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<th>Categories</th>
<th>Definition</th>
<th>Examples illustrating the meaning of the categorization</th>
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<td>Appropriate-</td>
<td>Coding rule: “MEAs’ correct response” (as ‘key concept’)</td>
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<td>Broad range of mathematical knowledge and abilities to produce a reusable and sharable conceptual tool.</td>
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<td>1. Knowledge</td>
<td>Students’ ability to utilize their prior and developed mathematical knowledge in various ways to develop an appropriate model.</td>
<td>The transcript (lines 1, 2 &amp; 1, 9) demonstrates how students apply their mathematical knowledge to construct (measure, code and synthesize) a relevant mathematical “object” such as their height and their shoe length, and mathematize the relationships between these “objects” to estimate their height (see also researcher’s interpretation in the third phase, Figure 4).</td>
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<td>2. Utility</td>
<td>Deliberate actions or means applied by students to generate useful solutions not only for the current situation, but for other similar situations as well (reusable).</td>
<td>In the transcript (lines 5, 7), students explain how they deliberately developed a useful conceptual mathematical tool to estimate the height of students in their group that could also be applicable to other students’ data (similar situations).</td>
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<td>3. Documentation</td>
<td>Students’ ability to apply varied representations to present and share information with others (shareable).</td>
<td>The students’ poster in Figure 4 shows how students used symbols, “drawing” and written explanations to mathematically communicate “how” they were actively attempting to make sense of the structured problematic “real-life” situations in a way that could be shareable with others.</td>
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<td>Mathematical \ Resourcefulness</td>
<td>Coding rule: “overcome difficulties” (as ‘key concept’)</td>
<td>Students’ ability to cope in a coherent and fluent manner and demonstrate flexible thinking involving consideration of different approaches or strategies to construct and elaborate a powerful conceptual tool.</td>
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<td>1. Fluency</td>
<td>Students’ tendency to consider or evaluate several ideas and perspectives.</td>
<td>The transcript (lines 1, 2) shows early stages of the students’ modeling process which involved fluent generation of different relevant mathematical objects, including shoe width, shoe length, shoe perimeter and student height, before an effective solution emerged.</td>
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<td>2. Flexibility</td>
<td>Students’ ease in switching from one mental operation to another, applying redefinition and transformation, and finding new ways to describe both the data set and its behavior.</td>
<td>In the transcript (lines 5–7), students describe how verifying their early conceptualization of the situation required further refinement that takes into account more “discovered” information and more relationships among the data that later describe their advanced interpretation, leading to the development of a more powerful mathematical model (Figure 4). This example reflects students’ ease in switching from one mental operation to another to describe both the dataset and its behavior via different types of representations.</td>
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<td>3. Elaboration</td>
<td>Students’ refinement, generalization and integrating abilities applied to developing a new level of more abstract or formal understanding.</td>
<td>The conceptual mathematical instrument demonstrated in Figure 4 and the transcribed explanation (lines 9–10) show how students elaborated (extended, refined and integrated) their ideas to develop a new level of more abstract or formal understanding and create a more generalized conceptual tool, as shown in the researcher’s mathematical interpretation in Figure 4.</td>
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<td>Inventiveness or Originality</td>
<td>To assign this category to the data, we looked for an appropriate and unique mathematical response in comparison to those developed by other groups (Guilford, 1967).</td>
<td>Students’ ability to break away from routine or bounded thinking to create unique and powerful mathematical ideas that differ from those developed by most other students.</td>
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<tr>
<td>1. Inventiveness</td>
<td>Students’ ability to break away from routine or bounded thinking to create unique and powerful mathematical ideas that differ from those developed by most other students.</td>
<td>The conceptual tool in Figure 4 illustrates students’ inventiveness. Although there were two other groups (out of 22) that estimated the individual’s height based on the ratio between height and the sum of shoe length and width, only this group used a split function to mathematically describe how an individual’s height depends on the width and length of his or her shoes.</td>
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The following contains a transcript of the poster presentation given by students A’ and S’; Figure 4 shows the students’ MEA documentation using WTS (Chamberlin, 2004) and the researcher’s (R’) mathematical interpretation of their work.

1. A’: "At the beginning we tried measuring only the length of each of our shoes, and then our height, but we couldn’t find any operation that led us to our height."
2. S’: "We measured the perimeter of our shoes but none of the operations we used led us to a reasonable height."
3. A’: "Then we measured the width of our shoes."
4. S’: "We tried width plus length multiplied by a whole number, for instance 5; for me it was right but for him [A’] it wasn’t. It was more than his height."
5. A’: "Then we noticed that my shoe is relatively wider and S’s shoe is narrow in comparison to its length."
6. S’: "So we decided that if the shoe in its narrowest part [pointing to his drawing] is less than 10 cm, we multiply it by 5. Otherwise we multiply by 4."
7. A’: "Then we tried it [their formula] on Y [member of another group] too."
8. R’: "I can see that you wrote A/S and erased the explanation you wrote in words."
9. S’: "We didn’t have time to complete our solution and find ways to describe the exact ratio so we compared the shoe's width to 10 cm and multiplied it by a fixed number, 4 or 5."
10. A’: "We wanted to use the proportion between length and width and to find a formula but we didn't have enough time for that so we just wrote A/S."

**DISCUSSION**

The results of this study indicate that engaging students in Model-eliciting activities (MEAs) not only provide students with the opportunity to apply and activate their Creative-Thinking abilities but also encourage their development and improvement (Lesh & Doerr, 2003; Gilat & Amit, 2012). Although the quantitative results are still preliminary and limited, it’s clearly indicates a positive effect of the MEAs intervention program on the development of students’ Creative-Thinking abilities. Furthermore the qualitative analytical process reveals how and what kind of mathematical Creative-Thinking abilities students applied and activated during the modeling process.
These results have both theoretical and practical implications (Amit, 2012; Gilat & Amit, 2012). In practice, they suggest new directions and alternatives for encouraging and inducing students to draw on those creative abilities more productively as suggested by Guilford (1950), who argued that creativity can be developed and the “development might be in the nature of actual strengthening of the functions involved or it might mean the better utilization of what resources the individual possesses, or both” (p. 448). Theoretically, it can provide us with a deeper insight into what is involved in the creative mathematical process of young students engaging in non-routine, “real-life”, structured problem-solving (Sriraman, 2009).

References


