Abstract
Horizon content knowledge (HCK) is one of the least understood domains of the Mathematical Knowledge for Teaching (MKT) framework. This paper reports from the Norwegian part of a research project seeking to obtain a deeper understanding on the nature and role of HCK and on how it can inform instruction. By using simulated teaching vignettes grounded in instructional episodes and interviews with teachers, prospective and practicing teachers were interviewed to understand the nature and content of their HCK better and to access and support teachers’ learning of HCK. Findings suggests that HCK seems to provide useful orientation to hearing and working with students’ ideas and that the simulated vignettes serve as a good avenue for thinking about and discussing HCK.

Introduction
Teachers’ content knowledge is important in teaching mathematics. Such knowledge can be perceived in different ways using various conceptualizations (Rowland, Huckstep, & Thwaites, 2005; Davis & Simmt, 2006). These different conceptualizations are all grounded in the work of Shulman and colleagues (Shulman, 1987; Wilson, Shulman, & Richert, 1987). Researchers have found that pedagogical content knowledge (Shulman, 1987) and specialized content knowledge (Ball, Thames, & Phelps, 2008) are related positively to student achievement gains (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Rockoff, Jacob, Kane, & Staiger, 2008). This type of content knowledge is different from the knowledge needed to solve mathematical problems in other professions where mathematics is needed. Ball et al. (2008) call such knowledge ‘Mathematical Knowledge for Teaching’ (MKT)—the mathematical knowledge, skills, and habits of mind used for the work of teaching.

Of the MKT domains, horizon content knowledge (HCK) is less developed and understood. HCK has been conceived as a domain that involves a sense of how mathematics used in instruction is related to the larger mathematical landscape (Ball & Bass, 2009). Many view HCK as knowledge about the mathematics outside the curriculum being taught, but most commonly, the knowledge about the curriculum that lies ahead of children have been investigated (Jakobsen, Thames, Ribeiro, & Delaney, 2012). In a working definition of HCK, Jakobsen et al. (2012) wrote:

HCK is an orientation to and familiarity with the discipline(s) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory. (p. 4642)

This article reports from the Norwegian part of a collaborative study involving prospective and practicing teachers from Cyprus, Norway, and Portugal (Charalambous, Jakobsen, & Ribeiro, 2013). Grounded in prior work looking for evidence of HCK in teachers’ practices (Jakobsen et al., 2012, 2013), we addressed the following questions:
1. What is the nature and role of HCK in informing instruction?
2. What is the potential of a designed environment using HCK vignettes in supporting teachers’ development of HCK?

Theoretical Background
Starting with Shulman’s (1987) categories subject matter knowledge (SMK) and pedagogical content knowledge (PCK) and using a practice based approach, Ball et al. (2008) at the University of Michigan developed a conceptualization of MKT. Ball et al. (2008) suggested dividing SMK and PCK into three distinct sub-domains—common content knowledge (CCK), specialized content knowledge (SCK), HCK—and indicated that MKT is a multidimensional construct.

Only the three sub-domains of SMK are discussed briefly in this article (see Ball et al. (2008) for a description of all domains). CCK is the domain corresponding to mathematical knowledge of fundamental importance in teaching, but people in other professions that commonly use mathematics also use it. SCK concerns the knowledge that allows the teacher to engage in tasks specialized to teaching—knowledge not needed in professions outside teaching. Examples of such knowledge are knowledge needed to generate representations and use them to explain core mathematical concepts and knowledge to analyze student errors and interpret nonstandard solution methods. In other words, CCK needs to be complemented by an understanding of how to make the content accessible to students, including knowing where and why students might encounter difficulties (Ribeiro & Carrillo, 2011).

HCK is the least developed and understood of these three sub-domains of SMK. Ball and Bass (2009) described HCK as “a kind of mathematical ‘peripheral vision’ needed in teaching, a view of the larger mathematical landscape that teaching requires” (p. 1). This is an attempt to name the more restricted way in which “advanced mathematics” is relevant to elementary school mathematics teaching. Many have perceived the meaning of “related to a larger mathematical landscape” in a broader way and this has opened different interpretations and views (see Jakobsen et al., 2013 for a discussion). In an attempt to name the way in which “advanced mathematics” can be relevant to school mathematics teaching, Jakobsen et al. (2012) developed a definition of HCK:

Horizon Content Knowledge (HCK) is an orientation to and familiarity with the discipline (or disciplines) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory. HCK includes explicit knowledge of the ways of and tools for knowing in the discipline, the kinds of knowledge and their warrants, and where ideas come from and how “truth” or validity is established. HCK also includes awareness of core disciplinary orientations and values, and of major structures of the discipline. HCK enables teachers to “hear” students, to make judgments about the importance of particular ideas or questions, and to treat the discipline with integrity, all resources for balancing the fundamental task of connecting learners to a vast and highly developed field. (p. 4642)

From this definition, it follows that HCK is neither common nor specialized and that it is not about a curriculum progression, but more about having a sense of the larger mathematical environment of the discipline being taught. Thus, when discussing HCK, it is not sufficient to simply consider knowledge about advanced mathematics (at a “more advanced level” than a teacher will have to teach) or knowledge about different topics that may arise in students’
future studies, as HCK also includes knowledge that would allow teachers to make sense of what students are saying and to act with an awareness of connections to topics that students may or may not meet in the future. Such an awareness, knowledge, and ability to deal with what is said—or not said—in the classroom are perceived as one key factor of HCK (Ball & Bass, 2009, Jakobsen et al., 2013).

**Methods**

We based our method on a practice-based approach (Cohen, Raudenbush, & Ball, 2003) in two ways. First, we relied on earlier work by Jakobsen et al. (2012), where teaching vignettes were designed from classroom events enriched with the analysis of discussions with practitioner teachers and other researchers. To establish their authenticity as realistic examples of teaching situations, teachers and professional educators validated the content of the vignettes. The second step of our exploration aimed at testing our hypotheses against empirical evidence. It is work related to this second step in Norway that this article documents.

In the second step, with the theoretical idea on vignettes and capitalizing on the work of Charalambous (2008), Charalambous et al. (2013) developed a teaching simulation to explore the extent to which a designed environment could better help access and understand the nature of teachers’ HCK, as well as provide a space for fostering teachers’ development of HCK. This teaching simulation consisted of a series of PowerPoint slides depicting a sixth-grade cartoon teacher delivering a lesson to an elementary grade class (see Figure 1 for sample slides). The lesson started with the teacher presenting the following problem to her sixth-grade students:

*Mark wants to make a pen for his new dog, Bozo. He has bought 20 meters of fence, and he wants to build a pen in the middle of his big garden. What shape of the pen would give Bozo most space to play? Justify your answer.*

The participating (prospective) teacher was first asked to solve Bozo’s problem themselves and to explain how they could be sure that the shape they found gave the most space for Bozo. Then they were asked to anticipate how sixth-graders would solve such a problem before they moved on to watch the animated teaching simulation.

After the cartoon teacher presented the problem to the class, the students solved the problem in pairs. The students explored ideas, such as triangles and different rectangles, including a square. After this work, there was a whole-class discussion, where pairs of students pointed out that they noticed that as the sides of the quadrilateral they constructed become equal in length, the area for Bozo’s pen increased. Another pair of students agreed that a $5 \times 5$ square gives the most space, with one student presenting this on the board. Keisha, another student, challenged this thinking by asking if perhaps another shape, not a quadrilateral, could give even more space to Bozo.

The simulation was stopped here and the participants were asked to explain how they would handle this situation and comment on what Keisha might have had in mind. They were also asked how they could build on her thinking to move the lesson further and to point to the important mathematical ideas that could be behind this question. After this, the simulation continued with two more slides where the same cartoon student proposed using a circle as a plausible shape that might give Bozo more area. Finally the participants were asked how they would handle such a proposal if they were the teacher.
Both the simulation and the accompanying interview protocols were first developed in English and then translated to Greek, Norwegian, and Portuguese.

**Results and Discussion**

Five prospective Norwegian teachers and one practicing teacher were interviewed. Three of the prospective teachers were in their first year of their teacher program and had completed a content and method course. The two others were towards the end of a two-year program in master education (taken on top of a teacher education) and had completed several mathematics and mathematics education courses. The teacher also had a master degree in mathematics education.

For the first part of the task—when the (prospective) teachers solved the problem by themselves—the answers could be grouped in three categories (Table 1):
Proposed solution | Number of participants, experience category
---|---
Circle | Two, one teacher and one master student, with 2–5 years’ experience.
Rectangle/Square | Two first-year students proposed rectangle at first, but concluded with square. One student had square at first.
Algebra/differentiate | One master student proposed using algebra first and then differentiating to find the shape that would give the greatest area.

Table 1: Proposed shapes/solution methods that would give the greatest area.

Both participants that answered that a circle was the solution based this on their experience. One expressed that he had seen something similar in a master course about the history of mathematics. They both hinted toward using a regular polygon to approximate a circle and that the area would increase as the number of edges increased toward infinity. For the four other participants, one said using algebra and derivation to find the shape that gave the greatest area, while two participants said rectangle but after some reflection, they concluded that a square would give the greatest area.

When asked how they anticipated sixth-graders would solve such a problem, all stated that students would suggest quadrilateral shapes. Participants with more experience expressed more ideas about how students could be challenged to come up with more shapes by asking them questions. Afterward, while observing the animation, one participant was surprised when a student proposed a triangle. This participant had limited her thinking to quadrilateral shapes only. Observing a triangle opened a new space of possible solutions and this participant later proposed circle as a solution.

Towards the end of the animation, when Keisha challenged the thinking and asked if another shape that was not a quadrilateral could give even more space to Bozo, all participants thought that she was thinking about a circle. Because of this development of their thinking while watching and reflecting upon the animation, it seemed that the animations supported the teachers’ learning of how to handle and make important judgments of particular ideas or questions and in that sense, supported the (prospective) teachers’ learning of what is part of HCK (Ball & Bass, 2009; Jakobsen et al., 2013). The ability to “hear” students’ ideas became more evident as the interview proceeded, but only two of the more experienced participants (both in terms of content/method courses and teaching) had ideas on how one could work with students in class. Knowing where ideas come from and of how truth and validity is established in mathematics is part of HCK (Ball & Bass, 2009; Jakobsen et al., 2009). Some participants had ideas on how regular polygons could be connected to a circle. One participant said “a circle is a regular polygon with indefinitely many sides” and proposed how students could work with regular polygons and calculate the area by using the formula for calculating the area of triangles.

These findings underscore the role of HCK as an integral component of the knowledge needed to teach mathematics, and especially to teach the subject in ways that enable rather than constrain future learning. It shows the potential of using simulated environments to support teachers’ development of HCK and preparing them for the complex work of teaching.

References


