On mathematics teaching in Finland

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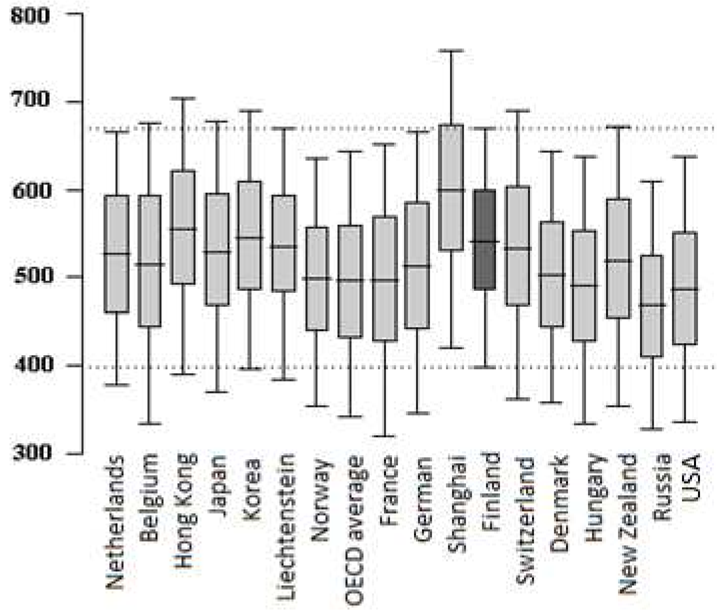
Abstract

Finnish math teaching has been highly praised due to the PISA survey, but Finnish mathematicians have criticized the news coverage of the success of the PISA-results. Recent studies show the deterioration of the level of math skills and claim that PISA-type problem solving is not the solution to basis for theoretical advancement. Among others the results of Näveri show that building a concrete model is necessary. In year 2000 an experiment to teach following the Hungarian Varga-Neményi -method was introduced in some voluntary primary Finnish schools. This method advances from concrete to abstract. In the presentation, details will be discussed.

PISA-success and its critics

Finland has received very good results in international PISA comparisons, yet many mathematics teachers worry about the level of mathematics learning in schools: Over 200 mathematics teachers in universities and polytechnics signed a statement (see Solmu) “The PISA survey tells only a partial truth of Finnish children's mathematical skills… Newspapers and media have advertised that Finnish compulsory school leavers are top experts in mathematics. However, mathematics teachers in universities and polytechnics are worried, as in fact the mathematical knowledge of new students has declined dramatically. As an example of this one could take the extensive TIMSS 1999 survey, in which Finnish students were below the average in geometry and algebra. This conflict can be explained by pointing out that the PISA survey measured only everyday mathematical knowledge, something which could be -_and in the English version of the survey report explicitly is – called "mathematical literacy"; the kind of mathematics which is needed in high-school or vocational studies was not part of the survey. No doubt, everyday mathematical skills are valuable, but by no means enough. The PISA assignments are simple numerical calculations, minor problems or deductions, interpretation of statistical graphics and evaluation of situations where text comprehension is an essential part. However, hardly any algebra or geometry is included. Nevertheless, the assignments are well in agreement with the goals of the survey; in fact, the goal was to study everyday mathematical knowledge. The PISA-survey leaves us, thus, with unanswered questions regarding many skills, like computing with fractions, solving elementary equations, making geometrical deductions, computing volumes of solid objects, and handling algebraic expressions. Still algebra is perhaps the most important subtopic in mathematical studies after the compulsory comprehensive school. In comprehensive school, the goal should be to learn the basic concepts of mathematics so that they can be used as a basis for more. Even the use of calculators does not change this situation: although calculators nowadays might be able to handle fractions, manual computation is essential to master since it is part of the foundations in handling algebraic expressions. Further study becomes impossible if the basics are not learned properly. One reason for the increase of poor standards in the matriculation exam and in the beginning of university studies is, undoubtedly, the weakness of the foundation received in the comprehensive school. New, more difficult concepts are hard to learn because still in upper secondary school much energy is spent in reviewing concepts that should have been learned in the comprehensive school. This vicious circle continues in tertiary education: the high-school concepts are not properly learned, and further learning becomes more difficult. The PISA survey provides us with useful information regarding the mathematical literacy needed in everyday life and the ability to solve simple problems. These skills are simply not enough in a world, which uses and utilizes mathematics more and more. A proper mathematical basis is needed especially in technical and scientific areas, biology included. The PISA survey tells very little about this basis, which should already be created in comprehensive school. Therefore, it would be absolutely necessary that, in the future, Finland would participate also in international surveys, which evaluate mathematical skills essential for further studies."



Picture 1 shows that the weakest Finnish students were the key to PISA 2009-success.

Collaboration with Hungary: Some Finns started looking for collaboration with some other country. A school system is like a pyramid, where each stage builds on the former ones. If some country produces good results on all levels up to the highest, it is probably due to a good tradition, especially if economical resources are scarce. In international comparisons Hungary has been such a country. The method chosen we call here Varga-Neményi method. It is very concrete, close to playing, in the beginning with a lot of use of manipulatives to support thinking and to create mental images. The goal is that all pupils finally leave their support. For some it happens faster, for some slower, especially if there are learning difficulties. Children enjoy this kind of activity, it helps them to learn to concentrate, and creates a positive attitude towards learning. Mathematical concepts are approached in a concrete manner, using different approaches for the same concept, in order to create a mental image, which will be made more exact later. If the basis is well constructed, it helps further learning. Symbols are only introduced after having been concretized in various ways. Mathematics skills are constructed like a house, basis of primary teaching is very important. In addition to mastering basic concepts, primary teaching influences strongly attitudes and studying techniques. Abstract mathematical concepts appear later on but have been introduced in a concrete manner in primary teaching. Hence they have time to "ripen".

Experiences in Finland: A lot of work has been done - and is still done in Finland training teachers, doing translations and adjusting the Hungarian materials into Finland. In order to be able to help create mental images of mathematical concepts for pupils one has to have the particular concept very clear in his or her own mind. The teachers have noticed that they need more mathematical knowledge and are eager to strengthen it. In year 2000 teaching first grade using the Varga-Neményi method was started in voluntary schools. It means extra work for teachers, but results are good. A solid basis is constructed, symbols, logic, concepts are focused on, no hurrying to big numbers, since learning difficulties often seem to start with poor number concept. This saves time later on, it is easier to absorb concepts on an abstract level when they have been carefully introduced on a concrete level. Teaching prepares for algebra from the beginning on. Calculators are included only when the necessary basis has been acquired. Special attention has been given to learning difficulties and teachers report that the method has been helpful. In Finland thousands of pupils are by now somehow influenced by the method. Voluntary teachers are each year trained to use the method. Many preschool teachers apply ideas of the method.

A typical difference between Hungarians and Finns appeared in a TIMMS-1999 assessment. Only 24% of Finnish 7.graders could solve the equation $12x-10 = 6x+32$ while 74% of Hungarians succeeded in this task. The average of all participating countries was 44%. To master manipulation with symbols, like x above, is essential for mathematics skills and one cannot proceed without it. In Hungarian teaching tradition preparation for first order equations starts right from 1st grade.

Algebraic thinking

Knowledge is either conceptual or procedural. Procedural knowledge includes the rules, algorithms and procedure. For example, the definition of the derivative is a conceptual concept, and when you derive, you use procedural knowledge. When a new concept is formed, the process is creative to the learner, hence cannot repeat the earlier model. The theoretical approaches to mathematical knowledge are formalism, Platonism and intuitionism. The aim of the intuitionist is to form a mental image. It should correspond with the concept as well as possible. Structurally, the formation of the concept is not the association chain, where one member produces the following. The concept and the concept image consist of all the cognitive structures in the individual's mind those are associated with a given concept (Tall & al. 1981). The symbol of the concept is a word, picture or symbol. However, one does not think with concepts, but with mental images and generalizations about concepts. When a child learns addition, first he/she learns with concrete pieces counting one by one. The strategies will be developed little by little and counting becomes automated. When the child sees an addition as an expression, which has an exact value, the child has moved from the procedural thinking to the conceptual thinking. For example, in arithmetic, the sum 3+4 has the exact answer 7. In algebra, by looking at the expression 3+x, Jockusch & McKnight (1978) found that one of the most difficult things to the pupils was to understand, what is the value of the expression 3+x. In algebra there is no analogous procedure with arithmetic.

Then, how can we learn algebra? For this, you should at the numerical level highlight structures, not results (Sfard 1995; Linchevski 1994). The pupil should observe the mathematical laws, regardless of the values used. Then structures become more important than numerical values and thinking in general level becomes possible. According to Sfard (1995) and Linchevski (1994) the development of algebraic structures should start at the level of numerical structures. If the basic arithmetical skills remain at the primitive procedural level, the conceptual thinking at the numerical level is not possible and the learning of algebra with understanding is difficult.

Table 1 shows a comparison between different levels of abstraction, tasks related to the basic calculations and analogous tasks with same structure, related to different expressions, in particular rational expressions (Näveri 2009). For example, the percentage of pupils who in the 1980´s solved the task 4/5 * 5 and the task 1/b * b was 24,2 % , in the 2000´s only 9,7 % . But for example the rational expression $x^{12}: x^4$ and the rational expression with same structure $15a^8 : 5a^4$ were solved in the 1980´s and 2000´s by almost the same percentage of pupils.

Table 1. Basic calculation divided into different levels of abstraction and analogous tasks. (percentage differences compared with the t-test). N = 763

	Different levels of abstraction				Analogous tasks	
	Addition %	Subtraction %	Multiplication %	Division %	Variables %	Fractions %
1980s	47,9	28,2	24,2	21,5	15,4	24,2
2000s	20,6***	15,1***	9,7***	7,3***	13,1	24,7

*p<0,05; **p<0,01 and ***p<0,001

There are pupils who don´t understand or have forgotten how to handle rational expressions, but they can reconstruct the structure from the corresponding numerical task and get support for their solutions (Schroeder & al. 1989).

In order to transfer a solution strategy at the concrete level to another context the memory hint needs to be close enough and the structures have to be similar. But if the general principle has been understood, it can be applied in different contexts (see picture 2).

Kaminski's (2008) study group reported in 2008 on a learning test, in which the students to whom mathematics had been taught up to the level of formal concepts, did always better than those to whom the corresponding subject had been taught using application tasks at a concrete level. The process of concept formation explains these variations: in the 1970s and 1980s, the goal in teaching was to go through the various phases of a procedure, so that the student could perform the same phases independently. The students learned by repeating the matter learned in a general form. They did not notice the connection between general forms and concrete situations. In contrast, when the starting point is specific and an encapsulated object level is enabled at a concrete and formal level, then deductive, backward-looking thinking becomes possible, and thinking at the general level can be applied to different contexts.

What does PISA measure?

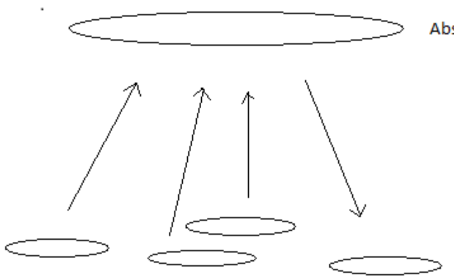
Finnish ninth-grade students participated in the PISA (Program for International Students Assessment) study program in 2000, 2003, 2006 and 2009. In the year 2000, mathematics was a minor area. In 2003, it was central. The goal of PISA 2003 was to evaluate the knowledge, skills, and readiness of young people from the viewpoint of future skill requirements.

Most of the PISA tasks show various kinds of diagrams or tables, and the required calculating operations were usually not complicated. The basic knowledge and skills of mathematics were defined as knowing terminology, fact knowledge, calculation and skills in using solution methods. These skills are of the basic level when evaluated with the taxonomy levels (Näveri 2009). They are not applicable, and thus do not create new knowledge. The taxonomy used is based on Wilson´s (1971) taxonomy.

In the literature the concept 'problem' has often been characterized as follows (e.g. Kantowski 1980): The task is said to be a problem, if the solution demands the solver to combine her/his earlier knowledge in a (for her/him) new way. When the solver immediately recognizes what she/he has to do in the task, it is said to be to her/him a routine task (or a standard task). The PISA 2003 was to study problem solving. This is as well the bases of the Finnish curriculum. In the PISA study, problem solving was defined as the individual ability to use cognitive processes in confronting and solving real problem tasks that cross subject boundaries, where the route leading to the solution is not immediately visible and where skill areas that may possibly be useful are not limited exclusively to the assessment areas of mathematics, natural sciences, or reading (Välijärvi 2004). Defined this way, problem solving was studied in the PISA study in tasks crossing subject boundaries, not in the context of mathematics. So the problem solution of PISA study is not directed to the context of mathematics.

Conclusion

“How ready are young people to meet the future challenges of everyday life, work, and lifetime learning? Do young people know how to look for and analyze information?” asks Välijärvi (2004) in his PISA-evaluation report. These are relevant questions. In the PISA study, as we saw, the basic knowledge and skills of mathematics were defined as knowing terminology, fact knowledge, calculation and skills in using solution methods, but operating on this basic level is not enough to use information and skills in new situations. As we saw above, the concrete level is necessary for basis to learn algebra. The picture below shows how pupil´s understanding on concrete level helps to recognize and use structures on formal level.



Through algebra, pupils learn to handle non-numeric symbols. Then, they can learn logical thinking, understanding and recognition of structures. For this reason, algebra is considered to be one of the most central areas of mathematics from the viewpoint of further studies after comprehensive school.

We have discussed here means to achieve critical, significant and flexible thinking, recognition of structures and apply information in new conditions, characteristics required of skilled people in the future.

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